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THE CYCLOID TRAJECTORY METHOD FOR WRAPING SURFACES ASSOCIATED OF ROLLING CENTROID STUDY.* I ALGORITHMS.

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ABSTRACT

The paper shows a model of expressing the wrapping condition which explaining in a specific way the relatives motion of the reference systems associated of wrapping centroids. We accept that point belongs of the engendering profile describes a family trajectory in relation with tool's centroids. The envelope of this trajectories is defined as the conjugate of engendering surface. Is shown a demonstration of equivalence of the new enveloping condition with Gohman theorem.

1. Introduction

Some method for reciprocal wrapping surfaces (contour) study are well know: the OLIVIER fundamental theorems, the kinematic method Gohman, the "normals" method (WILLIS), as well as the "minimum distance" method and the method of "substitutive circles" [1] [2].

Also, for reciprocal wraping tool forming of helical surfaces is known and used the NICOLAEV theorem.

The mentioned methods are characterized by specificity of applicability domain: generality for Olivier, with a high generality degree for Gohman method, of "minimum distance" and "substitutive circles family", particular for Willis and Nicolaev methods.

The specifics theorems of each method allows to solve same problem —the determination of characteristic curve (characteristic point) of two reciprocal wrapping surfaces, depending on contact kind between this —linear or punctiform.

The Olivier theorem, with theirs rigorously mathematical formulation, constitute the geometrical foundation of problem, reciprocal wrapping surfaces are examinate as deformable cloth depending on two variable parameters. In this way, the Olivier theorems allows the landing of any wrapping problem and no restricting —the immutable surfaces case— case that institute the application field of all methods.

Therefore the Gohman method represent a direct interpretation of Olivier theorems for the immutable reciprocal wrapping surfaces case, the specific theorems for others methods have enunciation characteristics and applicability forms which, formal, differentiate it.

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As follows, a new manner for examination of wrapping surfaces will be present, accepting a certain way to interpret the relative moving between centroids associated spaces.

2. The cycloid trajectory principle.

In fig.1 we present the centroids (circles, in figure) in rolling, as well as movement parameters φ_1 and φ_2.

In rolling movement of the centroids, the current point from Σ profile (profile which belongs of an vortex of profiles associated with one of the centroids) sketch a trajectory Ti of cycloid type in associated centroid space.

The ensemble of those trajectory determine a plane curves family of which envelope is determinate by point of view analytical.

Theorem:

The envelope of a profile associated with a centroid, belongs of a pair of rolling centroids is the envelope of the trajectory family sketch by those points in rolling centroid associated space.

It's clear that, dependent on the rolling centroids type (circles, circle and line) as well as the relative position of those, the trajectories can be epicycloid, hipocycloid or cycloid, for all these situations exists normal, lengthen or shorten shapes of the curves.

Next, will be present a demonstration for the theorem enunciation, as well as the application of these at engendering with gear rack, gear-shaped cutter and rotary cutter.

2.1. Gear rack engendering.

In fig 2, is show the ensemble of the rolling centroids as well as the reference system associate with these:

C_1-centroid associated of profile vortex to generate;

C_2-centroid associated of gear rack tool;

xyz-fixed reference system;

XYZ-mobile system, solidary of C_1 centroid;

ξηζ-mobile system, solidary of C_2 centroid.

The movement parameters λ and φ determine the rolling condition.:

$$\lambda = Rr \cdot \varphi. \qquad (1)$$

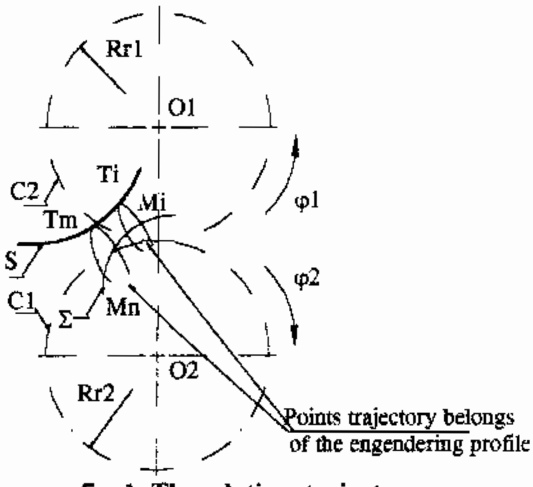

fig. 1. The relatives trajectory

The reference systems, solidary with the centroids, describes the absolutes movements:

$$x = \omega_3^T(\varphi) \cdot X \qquad (2)$$

which represent the movement of a point belongs of XYZ space given of xyz;

$$x = \xi + a;\ a = \begin{Vmatrix} -Rr \\ -Rr \cdot \varphi \\ 0 \end{Vmatrix} \qquad (3)$$

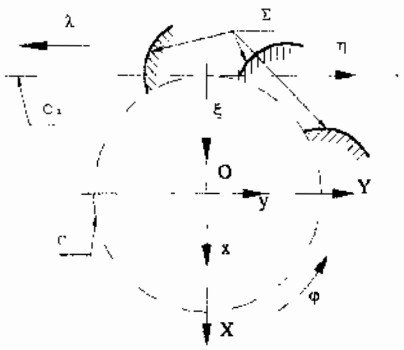

fig. 2. Gear tool engendering.

Equation (3) represent the system ξηζ trajectory, associate of gear rack tool given of fixed reference system.

The ensemble of absolute movements (2) and (3), determine the relative movement

$$\xi = \omega_3^T(\varphi)X - a \qquad (4)$$

Now, if in transformation (4), through X matrix we understand the geometric place of points belongs of Σ profile, in XYZ space, then after development the equation (4) can be look as represent equation of points trajectories belongs of Σ given the reference system of associated centroid.

Accepting that geometrical places of points belongs of Σ is in form

$$\Sigma:\ X = X(u);\ Y = Y(u) \qquad (5)$$

which represent a plane profile, with u variable, then from (4), through development we arrive at expression

$$\begin{Vmatrix} \xi \\ \eta \end{Vmatrix} = \begin{Vmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{Vmatrix} \cdot \begin{Vmatrix} X(u) \\ Y(u) \end{Vmatrix} - \begin{Vmatrix} -Rr \\ -Rr \cdot \varphi \end{Vmatrix}. \qquad (6)$$

After development, the transformation (6) represent the equations of the points trajectory belongs of Σ in space ξηζ,

$$(T_\Sigma)_\varphi \begin{vmatrix} \xi = X(u)\cos\varphi - Y(u)\sin\varphi + Rr; \\ \eta = X(u)\sin\varphi + Y(u)\cos\varphi + Rr \cdot \varphi. \end{vmatrix} \qquad (7)$$

Equation (7) represent, in principle equation of cicloidal type, in ξη space.

According to theorem, the envelope of these trajectory is the seeking profile —the gear rack tool.

The geometrical condition for determination of a curves family dependent of a parameter, in principle:

$$(T_\Sigma)_\varphi = \begin{vmatrix} \xi = \xi(u,\varphi) \\ \eta = \eta(u,\varphi) \end{vmatrix} \qquad (8)$$

is (9):

$$\frac{\xi'_u}{\xi'_\varphi} = \frac{\eta'_u}{\eta'_\varphi} \qquad (9)$$

Now, if we accept that Gohman [1], [2] condition in known

$$\vec{N}_\Sigma \cdot \vec{V}_\varphi = 0 \qquad (10)$$

where:

$\vec{N}_\Sigma$ is the normal at Σ profile, vector represent by (7), for φ=0;

$\vec{R}_\varphi$ is the speed vector in (7) movement.

In this way, the vectors are defined in the same reference system (ξη) and in same point (point which belongs of Σ profile).

Now, if we accept that, in principle the normal at Σ have the expression

$$\vec{N}_\Sigma = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \xi'_u & \eta'_u & 0 \\ 0 & 0 & 1 \end{vmatrix} = \eta'_u \cdot \bar{i} - \xi'_u \cdot \bar{j} \qquad (11)$$

and the $\vec{R}_\varphi$ vector, as speed vector (from (7) for u=cst.) have the leading parameters

$$\vec{R}_\varphi = \xi'_\varphi \cdot \bar{i} + \eta'_\varphi \cdot \bar{j} \qquad (12)$$

then, the Gohman condition (10) get form

$$\eta'_u \cdot \xi'_\varphi - \xi'_u \cdot \eta'_\varphi = 0 \qquad (13)$$

identical with condition (9), that was to demonstrate.
In consequence, the equation ensemble:

$$(T_\Sigma)_\varphi \left| \begin{array}{l} \xi = X(u)\cos\varphi - Y(u)\sin\varphi + Rr; \\ \eta = X(u)\sin\varphi + Y(u)\cos\varphi - Rr\cdot\varphi; \end{array} \right. \qquad \frac{\xi'_u}{\xi'_\varphi} = \frac{\eta'_u}{\eta'_\varphi}, \tag{14}$$

allowing to remove one of the two parameters, representing the trajectory family envelope $(T_\Sigma)_\varphi$, that is the gear rack tools profile —S_1 in principle in form

$$S \left| \begin{array}{l} \xi = \xi(\varphi) \\ \eta = \eta(\varphi) \end{array} \right. \tag{15}$$

2.2 Gear-shaped cutter engendering.

Similar, a solve of wrapping engendering can given if both centroids in rolling are circles —gear-shaped cutter engendering.

In figure 3, are present centroids and reference systems associated with.

The reference systems defined are:

xyz and $x_o y_o z_o$ fixed reference systems, with origins overlap of the centroids C_1 and C_2 centres;

XYZ the mobile system, solidary with Σ vortex (C_1 centroid);

ξηζ mobile system, solidary with gear-shaped cutter (C_2 centroid);

The engendering movements are rotation movements with parameters φ_1 and φ_2, between is the relation:

$$Rrp\cdot\varphi_1 = Rrs\cdot\varphi_2 \tag{16}$$

which is the rolling condition of the centroids.

The absolute movements of systems XYZ and ξηζ are given by transformations

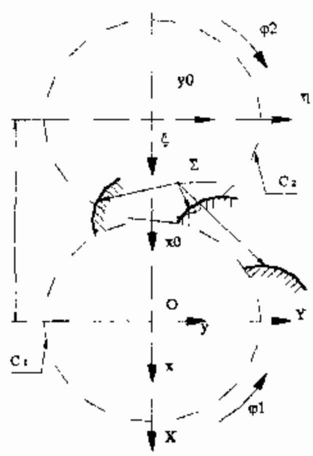

fig.3. Gear-shaped cutter engendering (external contact centroids).

$$x = \omega_3^T(\varphi_1)\cdot X \tag{17}$$

and

$$x_0 = \omega_3^T(-\varphi_2)\xi \tag{18}$$

In view of relative position of two fixed reference systems

$$x_0 = x - A;\ A = \left\| \begin{array}{c} -A12 \\ 0 \\ 0 \end{array} \right\| \tag{19}$$

We can define the relative movement in XYZ space confronted by ξηζ in form

$$\xi = \omega_3(-\varphi_2)\cdot\left|\omega_3^T(\varphi_1)\cdot X - A\right| \tag{20}$$

Now if in (20) the X matrix represent one matrix formed with amount of points belongs of XY space, which formed the Σ geometrical place

$$\Sigma : X = X(u);\ Y = Y(u) \tag{21}$$

with u variable, then, after the development, mind of (21) the matrix equation (20) will represent the trajectory family of points belongs to Σ profile given to ξηζ space —space associated with gear-shaped cutter:

$$(T_\Sigma)_{\varphi 1} \left| \begin{array}{l} \xi = X(u)\cdot\cos(1+i)\varphi_1 - \\ - Y(u)\cdot\sin(1+i)\varphi_1 + \\ + A_{12}\cdot\cos(i\varphi_1); \\ \eta = X(u)\cdot\sin(1+i)\varphi_1 + \\ + Y(u)\cdot\cos(1+i)\varphi_1 + \\ + A_{12}\cdot\sin(i\varphi_1), \end{array} \right. \tag{22}$$

were $i = \dfrac{\varphi_1}{\varphi_2}$ is transmission ratio.

Associating this equation with (9), where the partial derivation are given by (22), the equation ensemble (9), (22) represent the gear-shaped cutter, in form:

$$S \left| \begin{array}{l} \xi = \xi(\varphi_1); \\ \eta = \eta(\varphi_1), \end{array} \right. \tag{23}$$

Similar, the problem of internal contact of the gear-shaped cutter is solve.

In this case, the trajectory family is:

$$(T_\Sigma)_{\varphi 1} \left| \begin{array}{l} \xi = X(u)\cdot\cos(1-i)\varphi_1 - \\ - Y(u)\cdot\sin(1-i)\varphi_1 + \\ + A_{12}\cdot\cos(i\varphi_1); \\ \eta = X(u)\cdot\sin(1-i)\varphi_1 + \\ Y(u)\cdot\cos(1-i)\varphi_1 - \\ - A_{12}\cdot\sin(i\varphi_1), \end{array} \right. \tag{24}$$

equation which determine the $(T_\Sigma)_\varphi$ family envelope, that is the gear-shaped cutter profile for internal contact.

2.3 Rotary cutter engendering.

The reference systems keeps theirs semnifications (see fig.5).

The Σ profile is solidary with C_1 centroid, in translation motion.

$$x = X + a;\ a = \left\| \begin{array}{c} -Rr \\ -Rr\cdot\varphi \\ 0 \end{array} \right\| \tag{25}$$

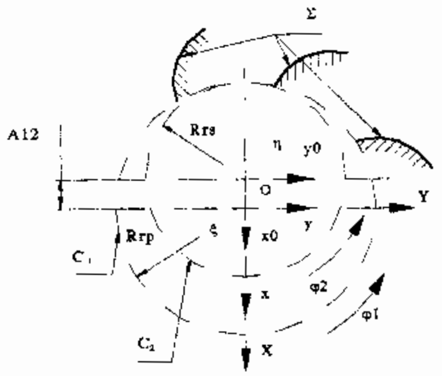

fig. 4. Gear-shaped cutter engendering (internal contact centroids).

The C_2 centroid and at the same time with, the rotary cutter make a rotary motion with angle φ as parameter, described by transformation

$$x = \omega_3^T(\varphi)\cdot\xi \qquad \textbf{(26)}$$

In this way, the relative motion of XYZ space given by ξηζ is

$$\xi = \omega_3(\varphi)[X + a] \qquad \textbf{(27)}$$

If in (27) through X matrix we understand the geometrical place of the point which represent the Σ profile, in form

$$\Sigma : X = X(u); Y = Y(u) \qquad \textbf{(28)}$$

with u variable, then after the development, the equation (27) represent the $(T_\Sigma)_\varphi$ trajectory family.

$$(T_\Sigma)_\varphi = \begin{vmatrix} \xi = [X(u) - Rrs]\cos\varphi - \\ - [Y(u) - Rrs\cdot\varphi]\cdot\sin\varphi; \\ \eta = -[X(u) - Rrs]\sin\varphi + \\ + [Y(u) - Rrs\cdot\varphi]\cdot\cos\varphi. \end{vmatrix} \qquad \textbf{(29)}$$

The $(T_\Sigma)_\varphi$ trajectory envelope is obtaining through equation (29) and (9), this envelope represent rotary cutter profile.

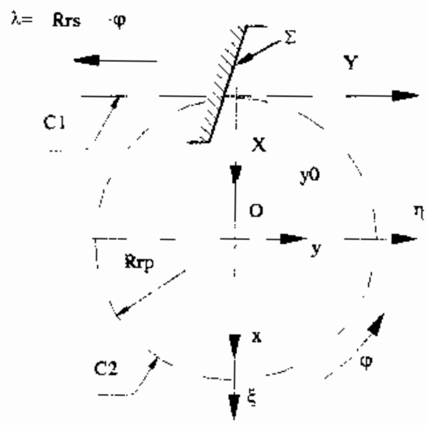

fig. 5. Rotary cutter engendering.

The gearing line, defined as geometrical place of contact points between the conjugated profiles in fixed reference systems, is defined in plane, associated at one of the absolute motions (of semi-product Σ or of tools S) the warping condition in form (9).

Thus, for gear rack tool, the gearing line is defined by equation ensemble:

$$LA\begin{cases} x = \omega_3^T(\varphi)X \\ \xi'_u\cdot\eta'_\varphi - \eta'_u\cdot\xi'_\varphi = 0 \end{cases} \qquad \textbf{(30)}$$

or in developed form:

$$LA\begin{cases} x = X(u)\cos\varphi - Y(u)\sin\varphi; \\ y = X(u)\sin\varphi - Y(u)\cos\varphi, \end{cases} \qquad \textbf{(31)}$$
$$\xi'_u\cdot\eta'_\varphi - \xi'_\varphi\cdot\eta'_u = 0,$$

where X(u), Y(u) are parametrical equations of Σ profile (5) in ξ(u,v), η(u,v) given by equation (7).

Similar, for rotary cutter engendering , the gear line is defined by equation ensemble:

$$LA\begin{cases} x = \omega_3^T(\varphi)X; \\ \xi'_u\cdot\eta'_{\varphi 1} - \eta'_u\cdot\xi'_{\varphi 1} = 0, \end{cases} \qquad \textbf{(32)}$$

keeping the anterior semnification of X matrix and consider ξ(u,v), η(u,v) given by (22) or (24), for internal rotary cutter engendering.

The gear line at rotary cutter engendering is defined by:

$$LA\begin{vmatrix} x = X + a \ \ (see\ 25) \\ \xi'_u\cdot\eta'_\varphi - \eta'_u\cdot\xi'_\varphi = 0 \ \ (see\ 9) \end{vmatrix} \qquad \textbf{(33)}$$

where ξ(u,v), η(u,v) are give by (29).

Note:

In case of Σ geometrical place expression in form of surfaces (cylindrical with generatrix parallel with Z axis), the gear line concept must be change with the gear surface concept. The X matrix, have in this case meanings that depend by two parameters, u and t.

$$\Sigma : X = X(u); Y = Y(u); Z = Z(t). \qquad \textbf{(34)}$$

Thus, the gear surfaces have equation in form.

$$SA\begin{vmatrix} x = x(u,\varphi); \\ y = y(u,\varphi); \\ z = t; \end{vmatrix} \qquad \textbf{(35)}$$
$$\xi'_u\cdot\eta'_\varphi - \eta'_u\cdot\xi'_\varphi = 0.$$

(See e.g. equation (31), for gear rack tool).

Contact lines.

The contact lines between the conjugate surfaces (characteristics) are defined as being the geometrical place of points belongs of conjugated surfaces, where those admits one common normal (the surfaces are tangents).

In this way, in gear rack engendering, if Σ surface, cylindrical, have equation in form (34) so from (3), result the Σ surfaces family in form:

$$(\Sigma)_\varphi\begin{vmatrix} \xi = \xi(u,\varphi) \\ \eta = \eta(u,\varphi) \\ \zeta = t \end{vmatrix} \qquad \textbf{(36)}$$

The contact line (the tangent line between Σ and S — the gear rack flank) is obtain associating with (36) equations the wraping condition (9) and the condition φ=cst.

So, the contact line —the characteristic— have the equation:

$$L_{\Sigma,S}\left|\begin{array}{l}\xi=\xi(u,\varphi);\\ \eta=\eta(u,\varphi);\\ \zeta=t;\end{array}\right.\qquad(37)$$
$$\frac{\xi'_u}{\xi'_\varphi}=\frac{\eta'_u}{\eta'_\varphi};$$
$$\varphi=const.(\varphi=0)$$

Note:

1. In case when Σ is a cylindrical surface, the contact line $L_{\Sigma,S}$ is a straight line parallel to Σ surface generatrix.

2. In case when Σ is a plane profile, the (37) equations are reduced to a point —the contingence point of the conjugated profiles:

$$M_{\Sigma,S}\left|\begin{array}{l}\xi=\xi(u,\varphi);\\ \eta=\eta(u,\varphi);\\ \xi'_u\cdot\eta'_\varphi-\eta'_u\cdot\xi'_\varphi=0;\\ \varphi=const.\end{array}\right.\qquad(38)$$

3. Similar are defined this concepts at gear-shaped cutter engendering.

Conclusions:

The way to interpret the relative motion between the associated surfaces (profiles) of some pairs of rolling centroids as determining trajectory family of points belongs of Σ profiles reducing the problem at a plane problem (in transversal plane of the generatrix cylindrical surfaces) allow to use one geometrical enveloping condition (condition (9))

The equivalence between condition (9) and Gohman condition was demonstrate.

The enveloping condition have the advantage of a simple expression, easy to remember and especially easy to use.

All the more, the graphic representation of the trajectory family can eliminate in a lot of practical situations the errors due firstly of the singulars points belongs of wrapping surfaces.

4. References.

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Metoda traiectoriilor cicloidale pentru studiul suprafețelor în înfășurare asociate unor centroide în rulare. I-Algoritmi.

Rezumat.

Se prezintă o modalitate de exprimare a condiției de înfășurare, interpretând, într-un mod aparte mișcările relative ale sistemelor de referință asociat unor cupluri de centroide în rulare. Se acceptă că puncte aparținând profilului de generat descriu o familie de traiectorii în raport cu centroida sculei. Înfășurătoarea acestor traiectorii se definește ca profilul conjugat suprafeței de generat. Se prezintă o demonstrație a echivalenței noii condiții de înfășurare cu teorema Gohman.

La méthode de les trajectoires pour l'étude de les surfaces en enveloppement associé a centroids en rouler. I Algorithms.

Resumé.

Se présente une modalité d'exprimer la condition d'enveloppement, interpréter, d'une maniere différente les mouvement relatives de les systémes de référence associe avec couples de centroides en rouler.

On accepte que des points qui appartiens d'une profile d'engendrement décrit une famille de trajectoires par rapport á centroid de outils. L'enveloppe de ces trajectoires est défine comme le profile conjugué á sourfaces d'engendrement.

Se presente une demonstration de l'equivalence de cet théoréme avec théoréme Gohman.